Heavy quarkonium in a weakly-coupled QGP using EFTs

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Work done in collaboration with N. Brambilla, J. Ghiglieri, J. Soto and A. Vairo.

Outline

1. Motivation
2. The $\frac{1}{r} \gg T \gg \Delta E \gg gT$ regime
3. $m \gg T \gg \frac{1}{r} \sim gT$. Dissociation temperature.

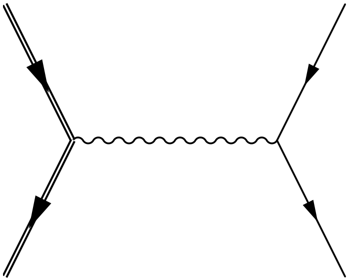
- Motivation

Energy scales for zero temperature heavy quarkonium

Heavy quarkonium at $T = 0$ is a system with a lot of different energy scales.

For example, for computing the decay of J/Ψ to electrons...

- We need annihilation cross section of the quark and the anti-quark to electrons. The energies involved are of the order of m_c.



Energy scales for zero temperature heavy quarkonium

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For example, for computing the decay of J/Ψ to electrons...

- We also need the probability that the quark and the anti-quark are at the same point, this is given by the wave-functions. The energies involved are of the order of $1/r$.

$$\Psi_{ab}(\mathbf{r})$$

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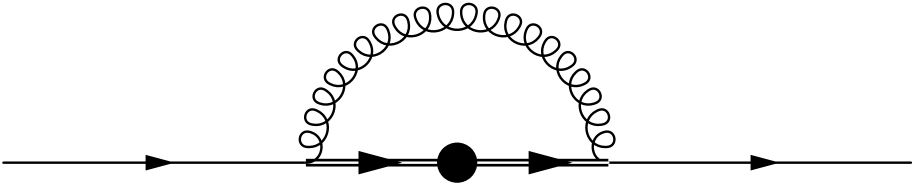

Energy scales for zero temperature heavy quarkonium

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For example, for computing the decay of J/Ψ to electrons...

- We need annihilation cross section of the quark and the anti-quark to electrons. The energies involved are of the order of m_c.
- We also need the probability that the quark and the anti-quark are at the same point, this is given by the wave-functions. The energies involved are of the order of $1/r$.
- If we want to make a precision computation, we need to include the effects of the color octet component of J/Ψ. The energy involved here is of order of the binding energy.

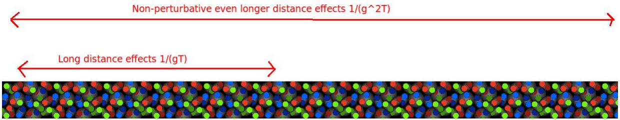
Energy scales in a thermal bath

In the weak-coupling regime

- We have an almost free gas of quarks and gluons with typical energy πT.
- At long distances (order $\frac{1}{gT}$), non-trivial collective phenomena arise, as for example chromoelectric static fields screening.
- At even longer distances (order $\frac{1}{g^2 T}$), non-perturbative phenomena arise, as for example chromomagnetic static fields screening.

Energy scales in a thermal bath

In the weak-coupling regime



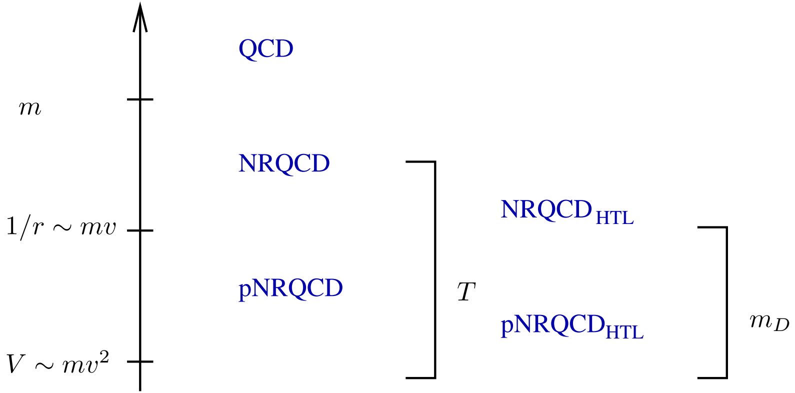
Heavy quarkonium in a thermal bath

In this case we have to combine the two types of energy scales.

- The energy scales typical of a non-relativistic bound state. m_Q, $\frac{1}{r}$ and ΔE.
- The energy scales of a weakly-coupled quark-gluon plasma. πT, gT...

Depending on the relation of T with the energy scales of the bound state we are going to have very different physical situations.

Effective field theories



Applications of theses EFTs to non-relativistic bound states at finite temperature

Previous applications

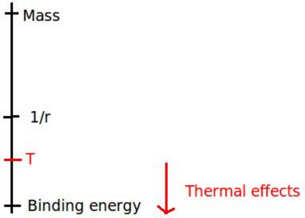
- Static limit of heavy quarkonium.
  N. Brambilla, J. Ghiglieri, P. Petreczky and A. Vairo.
- Hydrogen atom.
  M.A.E and J. Soto.
- Muonic hydrogen.
  M.A.E and J. Soto.

Now we want to study a more realistic situation.

- Energy spectrum and decay width up to $m\alpha_{\mathrm{s}}^5$ precision for $\frac{1}{r} \gg T \gg \Delta E \gg gT$.
- Leading order thermal corrections for $m_Q \gg T \gg \frac{1}{r} \sim gT$, and dissociation temperature for $1S$ charmonium and bottomonium.

- The $\frac{1}{r} \gg T \gg \Delta E \gg gT$ regime

Thermal effects



- The physical results that come from energy scales higher than the temperature are not affected by the thermal bath.

$$\frac{1}{e^{q/T} \pm 1}$$

- Consequence: The EFT resulting from integrating out degrees of freedom higher than T are still valid for this situation.
- These are NRQCD and pNRQCD.

NRQCD

- Gives exactly the same results as QCD for all Green functions evaluated at distances bigger than $\frac{1}{m_Q}$.
- One can always compute the NRQCD from QCD (matching) using perturbation theory because $m_Q \gg \Lambda_{QCD}$.
- This EFT is also useful for Lattice computations as the UV cutoff of this theory is much smaller than m_Q.
- Suppressions in $\frac{1}{m_Q}$ that are not obvious from QCD are trivially seen with this Lagrangian.

NRQCD

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{d_2}{m_Q^2}F^a_{\mu\nu}D^2F^{\mu\nu a} + d_g^3\frac{1}{m_Q^2}gf_{abc}F^a_{\mu\nu}F^{\mu b}_\alpha F^{\nu\alpha c}$$

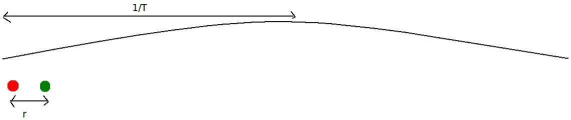
$$\mathcal{L}_\psi = \psi^\dagger\left(iD_0 + c_2\frac{\mathbf{D}^2}{2m_Q} + c_4\frac{\mathbf{D}^4}{8m_Q^3} + c_F g\frac{\sigma\mathbf{B}}{2m_Q} + c_D g\frac{\mathbf{DE}-\mathbf{ED}}{8m_Q^2}\right.$$
$$\left. + ic_S g\frac{\sigma(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_Q^2}\right)\psi$$

$$\mathcal{L}_\chi = c.c \text{ of } \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1({}^1S_0)}{m_Q^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{f_1({}^3S_1)}{m_Q^2}\psi^\dagger\sigma\chi\chi^\dagger\sigma\psi + \frac{f_8({}^1S_0)}{m_Q^2}\psi^\dagger T^a\chi\chi^\dagger T^a\psi$$
$$+\frac{f_8({}^3S_1)}{m_Q^2}\psi^\dagger T^a\sigma\chi\chi^\dagger T^a\sigma\psi$$

pNRQCD

There are still simplifications that are not obvious from NRQCD.



Thermal effects are going to see the bound state as a color dipole.

pNRQCD

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left. + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)\,Tr(O^\dagger \mathbf{r} g\mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O)$$
$$+ \frac{V_B(r)}{2}\,Tr(O^\dagger \mathbf{r} g\mathbf{E} O + O^\dagger O \mathbf{r} g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Gives the same results as QCD and NRQCD for Green functions evaluated at distances much bigger than r.
- The matching between NRQCD and pNRQCD can be done perturbatively if $\frac{1}{r} \gg \Lambda_{QCD}$.
- The degrees of freedom for the heavy quarks are now a color singlet field and a color octet field.
- Using pNRQCD provides automatically with a Coulomb resummation.

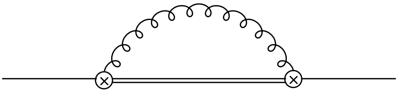
From *pNRQCD* to $pNRQCD_{HTL}$

Now we take into account thermal effects. For this we integrate out degrees of freedom with virtuality of order T^2 and we go from *pNRQCD* to a new EFT $pNRQCD_{HTL}$.

- In the gluons and light quarks sector of $pNRQCD_{HTL}$ we will have the usual Hard Thermal Loop action.
- The potential of the singlet and the octet will have thermal corrections

Integrating out the T scale, Leading order in α_s

Now we take into thermal effects into the singlet potential



$$-ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}[k_0^2 D_{ii}(k_0,k)+k^2 D_{00}(k_0,k)]r^i$$

All the thermal bath information is encoded in the gluon propagator D.

Integrating out the T scale, Leading order in $\alpha_{\rm s}$

Because $T \gg E$ we must expand the octet propagator

$$\frac{1}{E-h_o-k_0+i\eta} = \frac{1}{-k_0+i\eta} - \frac{E-h_o}{(-k_0+i\eta)^2} + \cdots$$

It is a polynomial in $(E-h_o)$

$$E-h_o = E-h_s-(V_o-V_s) = E-h_s-\Delta V$$

- If we only take into account $\Delta V = \frac{N_c\alpha_{\rm s}}{2r}$ we are doing the static limit.
- If we only take into account $E-h_s$ up to trivial color factors we are doing the QED case.

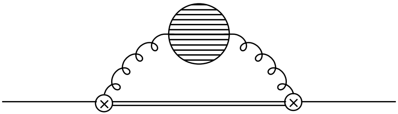
Integrating out the T scale, Leading order in $\alpha_{\rm s}$

$$\delta V_s = \frac{\pi}{9} N_c C_F \,\alpha_{\rm s}^2 \, T^2 \, r + \frac{2\pi}{3m} C_F \,\alpha_{\rm s} \, T^2$$
$$+\frac{\alpha_{\rm s} C_F I_T}{3\pi}\left[-\frac{N_c^3}{8}\frac{\alpha_{\rm s}^3}{r} - \frac{N_c(N_c+2C_F)\alpha_{\rm s}^2}{mr^2} + \frac{4(N_c-2C_F)\pi\alpha_{\rm s}}{m^2}\delta^3(\mathbf{r}) + N_c\frac{\alpha_{\rm s}}{m^2}\left\{\nabla_{\mathbf{r}}^2, \frac{1}{r}\right\}\right]$$

There is an infrared divergence

$$I_T = \frac{2}{\epsilon} + \ln\frac{T^2}{\mu^2} - \gamma_E + \ln(4\pi) - \frac{5}{3}$$

Integrating out the T scale, next to leading order in $\alpha_{\rm s}$



$$\delta V_s^{(2\,\mathrm{loops})} = -\frac{3}{2}\zeta(3)\, C_F\, \frac{\alpha_{\rm s}}{\pi}\, r^2\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_{\rm s}^2\, r^2\, T^3$$
$$+i\left[\frac{C_F}{6}\alpha_{\rm s}\, r^2\, T\, m_D^2\, \left(-\frac{2}{\epsilon} + \gamma_E + \ln\pi - \ln\frac{T^2}{\mu^2} + \frac{2}{3} - 4\ln 2 - 2\frac{\zeta'(2)}{\zeta(2)}\right)\right.$$
$$\left. + \frac{4\pi}{9}\ln 2\, N_c C_F\, \alpha_{\rm s}^2\, r^2\, T^3\right]$$

This contribution was first found in the static limit by Brambilla, Ghiglieri, Petreczky and Vairo.

Computations in the $m\alpha_s^2$ scale

- Because $T \gg m\alpha_s^2$

$$\frac{1}{e^{\beta k}-1} \to \frac{T}{k} - \frac{1}{2} + \cdots$$

- For gluons with $k_0 \sim m\alpha_s^2$ and virtuality $\Lambda^2 \sim (m\alpha_s^2)^2$ the HTL Lagrangian has to be taken into account but does not need to be resummed. We call this the off-shell region.
- For gluons with $k_0 \sim m\alpha_s^2$ but virtuality $\Lambda^2 \lesssim m_D^2$ the HTL Lagrangian has to be resummed. We call this the collinear region.
- The results coming from this energy scale can not be reproduced by a potential.

Contribution from the $m\alpha_{\rm s}^2$ scale

$$\delta E_{n,l} = -\frac{\pi\alpha_{\rm s} C_F \; T m_D^2}{3}\frac{a_0^2 n^2}{2}[5n^2+1-3l(l+1)]$$

a_0 is the Bohr radius of the fundamental state.

$$\begin{aligned}\delta\Gamma_{n,l} &= \tfrac{1}{3}N_c^2 C_F\alpha_{\rm s}^3 T - \tfrac{16}{3m}C_F\alpha_{\rm s} T E_n + \tfrac{8}{3}N_c C_F\alpha_{\rm s}^2 T\tfrac{1}{mn^2 a_0} \\ &+\tfrac{2E_n\alpha_{\rm s}^3}{3}\left\{\tfrac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left(\tfrac{8}{n(2l+1)} - \tfrac{1}{n^2} - \tfrac{2\delta_{l0}}{n}\right) + \tfrac{2N_c^2 C_F}{n(2l+1)} + \tfrac{N_c^3}{4}\right\} \\ &-\tfrac{\alpha_{\rm s} C_F T m_D^2}{6}\left(\tfrac{2}{\epsilon} + \ln\tfrac{E_1^2}{\mu^2} + \gamma_E - \tfrac{11}{3} - \ln\pi + \ln 4\right) a_0^2 n^2[5n^2+1-3l(l+1)] \\ &+\tfrac{2\alpha_{\rm s} C_F T m_D^2}{3}\tfrac{C_F^2\alpha_{\rm s}^2}{E_n^2}\, I_{n,l}\end{aligned}$$

$$I_{n,l} = \frac{E_n^2}{C_F^2\alpha_{\rm s}^2}\int\frac{d^3k}{(2\pi)^3}\,|\langle n,l|\mathbf{r}|\mathbf{k}\rangle|^2\ln\frac{E_1}{E_n - k^2/m}$$

Contribution from the $m\alpha_s^2$ scale. Comments

At $T = 0$ the c_D coefficient from NRQCD has an IR divergence.
$\delta E^{US}|_{T=0}$ has an UV divergence.
Both cancel out.

At finite T some of the corrections to the potential in $pNRQCD_{HTL}$ can be encoded in a correction to c_D so that it is IR safe. The V_s potential has an IR divergence proportional to T^3.
$\delta E^{US} = \delta E^{US}|_{T=0} + \delta E^{US}|_T$ has only an UV divergence proportional to T^3.
Both cancel out.

Final result. $m\alpha_s \gg T \gg m\alpha_s^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_s^2$ scale are subtracted).

$$
\begin{aligned}
\delta E_{n,l}^{(\mathrm{thermal})} = {} & \frac{\pi}{9} N_c C_F \, \alpha_s^2 \, T^2 \frac{a_0}{2} \left[3n^2 - l(l+1)\right] + \frac{\pi}{3} C_F^2 \, \alpha_s^2 \, T^2 \, a_0 \\
& + \frac{E_n \alpha_s^3}{3\pi} \left[\log\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right] \left\{\frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] \right. \\
& \left. \qquad + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\} \\
& + \frac{2E_n C_F^3 \alpha_s^3}{3\pi} L_{n,l} \\
& + \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right] \left\{ - \left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F \, \alpha_s \, T \, m_D^2 \right. \\
& \left. \qquad + \frac{2}{3}\zeta(3) \, N_c C_F \, \alpha_s^2 \, T^3 \right\}
\end{aligned}
$$

where $E_n = -\frac{mC_F^2\alpha_s^2}{4n^2}$, $a_0 = \frac{2}{mC_F\alpha_s}$ and $L_{n,l}$ is the Bethe logarithm.

Final result. $m\alpha_s \gg T \gg m\alpha_s^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_s^2$ scale are subtracted).

For a general number of colors N_c ($C_F = (N_c^2-1)/(2N_c)$):

$$\delta E_{n,l}^{(\mathrm{thermal})} = \frac{\pi}{9} N_c C_F\, \alpha_s^2\, T^2 \frac{a_0}{2}\left[3n^2 - l(l+1)\right] + \frac{\pi}{3} C_F^2\, \alpha_s^2\, T^2\, a_0 \qquad \sim m\alpha_s^5 \frac{T^2}{E^2}$$

$$m\alpha_s^5 \sim \begin{cases} +\dfrac{E_n \alpha_s^3}{3\pi}\left[\log\left(\dfrac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right]\left\{\dfrac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\dfrac{8}{n(2l+1)} - \dfrac{1}{n^2} - \dfrac{2\delta_{l0}}{n}\right]\right. \\ \qquad\qquad \left. + \dfrac{2N_c^2 C_F}{n(2l+1)} + \dfrac{N_c^3}{4}\right\} \\ +\dfrac{2E_n C_F^3 \alpha_s^3}{3\pi} L_{n,l} \end{cases}$$

$$m\alpha_s^6 \frac{T^3}{E^3} \sim +\frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3l(l+1)\right]\left\{-\left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F\, \alpha_s\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_s^2\, T^3\right\}$$

where $E_n = -\dfrac{m C_F^2 \alpha_s^2}{4n^2}$, $a_0 = \dfrac{2}{m C_F \alpha_s}$ and $L_{n,l}$ is the Bethe logarithm.

Final result. $m\alpha_s \gg T \gg m\alpha_s^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_s^2$ scale are subtracted).

$$
\begin{aligned}
\Gamma_{n,l}^{(\text{thermal})} = & \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3 T}{n^2}(C_F+N_c) \\
& + \frac{2E_n\alpha_s^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\} \\
& - \left[\frac{C_F}{6}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\, N_c C_F\, \alpha_s^2\, T^3\right] \\
& \qquad \times a_0^2 n^2\left[5n^2 + 1 - 3l(l+1)\right] \\
& + \frac{8}{3}C_F\alpha_s\, T m_D^2\, a_0^2 n^4\, I_{n,l}
\end{aligned}
$$

Final result. $m\alpha_s \gg T \gg m\alpha_s^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_s^2$ scale are subtracted).

$$
m\alpha_s^5 \sim \boxed{\begin{aligned}\Gamma_{n,l}^{(\mathrm{thermal})} = \; & \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2 \alpha_s^3 T}{n^2}(C_F + N_c) \;\sim m\alpha_s^5 \frac{T}{E} \\ & + \frac{2E_n \alpha_s^3}{3}\left\{\frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\}\end{aligned}}
$$

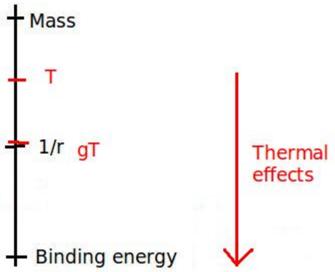
$$
m\alpha_s^6 \frac{T^3}{E^3} \sim \left\{ \boxed{\begin{aligned} & -\left[\frac{C_F}{6}\alpha_s T m_D^2 \left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\, N_c C_F\, \alpha_s^2\, T^3\right] \\ & \qquad\qquad\qquad\qquad \times a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1)\right] \\ & + \frac{8}{3} C_F \alpha_s\, T m_D^2\, a_0^2 n^4\, I_{n,l} \end{aligned}} \right.
$$

Conclusions

- For this temperature we can reach a high precision. Order $m\alpha_s^5$ in the energy and decay width.
- This can be interesting for bottomonium. Above the dissociation temperature of charmonium but below the one of bottomonium.
- This temperature is above T_c so perturbation theory might be reliable.

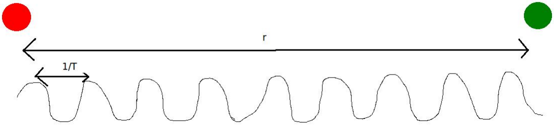
- $m \gg T \gg \frac{1}{r} \sim gT$. Dissociation temperature.

Thermal effects



Now we can start with NRQCD Lagrangian at $T = 0$. Thermal effects can be included in a new EFT called $NRQCD_{HTL}$.

Thermal effects



- Effects at the energy scale T are going to see heavy quarks as elements that are very far away from each other.
- We will have to average this thermal fluctuations in the gluons that are interchanged by the heavy quarks.

$NRQCD_{HTL}$

- We will have HTL corrections in the gluon and light quarks propagators. In fact, this is going to be the more relevant change for bound states phenomenology.
- NLO corrections to heavy quark sector.

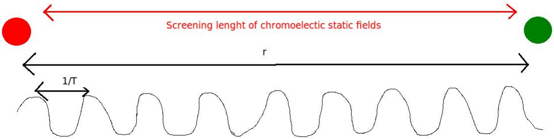
$NRQCD_{HTL}$

- We will have HTL corrections in the gluon and light quarks propagators. In fact, this is going to be the more relevant change for bound states phenomenologically.
- NLO corrections to heavy quark sector.

Why is it NLO?

- In the Coulomb gauge at the scale T, only the spatial gluons are thermalized: A_i.
- In this gauge A_0 is not modified by the temperature at LO up to the scale gT.
- The coupling of heavy quarks with A_i is always multiplied at least by one power of $\frac{1}{m_Q}$.

$NRQCD_{HTL} \to pNRQCD_{HTL}$



- After averaging thermal fluctuations we find HTL propagator.
- This HTL modification is a leading order effect in the A_0 field at distances r because $\frac{1}{r} \sim m_D$.
- The LO potential is going to be modified, and this can break the bound state.

The potential at $m \gg T \gg m\alpha_s$

$$V(r) = -\frac{\alpha_s e^{-m_D r}}{r} - \alpha_s m_D$$
$$+\frac{i16\alpha_s^2 C_F T^3}{\pi m_D^2}\left(\frac{\pi^2 (m_D^{lq})^2}{4T^2 g^2} + g(m_c\beta) + \frac{m_c^2}{2T^2(e^{\beta m_c}+1)}\right)\phi(m_D r),$$

- We consider the effect of the charm mass that can be important for bottomonium. The $m_c \to 0$ result was first found by Laine, Philipsen, Romatschke and Tassler.
- m_D^{lq} is the Debye mass that is found in the $m_c \to \infty$ limit.
- $g(0) = \frac{\pi^2}{12}$ and goes exponentially to zero at large values of $m_c\beta$.
- $$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[\frac{sin(zx)}{zx} - 1\right]$$

The dissociation temperature

- The temperature when the imaginary part of the potential is of the same order of magnitude as the real part is lower than the temperature where screening is important for bound states (in the limit of weak coupling).
- If both screening and dissipation are a perturbation the bound state survives.
- If the imaginary part of the potential is bigger than the real part then the the bound state does not exist anymore.
- It is natural then to propose as dissociation temperature the temperature where the real part of the potential is as big as the imaginary part.

$$\frac{1}{a_0^3} = 16\alpha_{\rm s}(\pi T) C_F T^3 \left(\frac{\pi^2 (m_D^{lq})^2}{4T^2 g^2} + g(m_c\beta) + \frac{m_c^2}{2T^2(e^{\beta m_c}+1)} \right).$$

Dissociation temperature for charmonium

Assuming g small and $\frac{1}{a_0} \gg m_D$. For J/Ψ

$\alpha_{\rm s}$	T_d (MeV)
$\alpha_{\rm s}(\pi T)$	230
$\alpha_{\rm s}(2\pi T)$	280

Slightly above what is found on the Lattice.
For example, A. Mocsy and P. Petrecsky found using a potential based on Lattice data

$$T_d \leq 1.2 T_c$$

In Tuesday H. Satz talk it was $T_d = 1.5 - 2.0\,T_c$, and $T_c \sim 165 - 195 MeV$ according to QWG paper of 2010.

Dissociation temperature for bottomonium

For $\Upsilon(1S)$

$\alpha_{\rm s}$	T_d (MeV)
$\alpha_{\rm s}(\pi T)$	440
$\alpha_{\rm s}(2\pi T)$	500

Compatible with recent Lattice calculations.
For example, Aarts et al. in 2010 found that $T_d > 2.1 T_c$.

Dependence of bottomonium dissociation with charm mass

m_c (MeV)	T_d (MeV)
∞	480
5000	480
2500	460
1200	440
0	420

Table: Dissociation temperature for Upsilon (1S) for different values of the charm mass

Conclusions

- The dissociation temperature if found when the temperature is much higher than the inverse of the typical radius.
- In the limit of $g \to 0$ the dominant dissociation mechanism is the imaginary part of the potential.
- The results found in this approximation are not very far away from Lattice results, although g is not so small.
- The use of EFTs allow to make computations in a very systematic way. It makes easier to see what simplifications are allowed (for example multipole expansion for $\frac{1}{r} \gg T$), and also when a resummation (for example HTL) is needed.
- Moreover, they provide tools to know the size of a contribution before computing it.